Names____
Section ________

PHYSICS 315 –COMBAT AVIATION PHYSICS

Application Exercise 2

Why Do I Need Phasors to Fly the Raptor and Kill the Enemy?

Due: ***Beginning of class*****, lesson 21**

100 points

To receive full credit you must show all work and communicate efficiently using proper grammar.

AUTHORIZED RESOURCES: *any published or unpublished sources and any individuals*.

Document appropriately!

Why Do I Need Phasors to Fly the Raptor and Kill the Enemy?

Note: you can find a color version of this handout on the website.

A quick glance at the remaining chapters of *Stimson* should convince you that phasor diagrams will play a prominent role. The huge benefit of a phasor representation of a signal is that you can look at pictures and completely understand signal processing concepts as well as you would from a rigorous mathematical treatment. Trust me: fighter pilots like pictures much more than math.

In this computer exercise, we'll examine how two sinusoidal signals are mixed together, and how important data about an unknown signal can be retrieved from this mixture. First, you need to read the attached Mathcad document entitled "How Phasors Make Signal Processing Understandable." It *complements* Chapter 5 in *Stimson*, "Key to a Nonmathematical Understanding of Radar."

Next, you will find a Mathcad file called "Application Exercise 1.mcd" loaded onto the computers in the back of the room. Open the file and <u>save it to your own disk</u>. It contains the exact formulae and plots from the attached printout without all of the lucid prose. A more detailed form of that file is attached to these instructions.

Your goals for this exercise are as follows:

1. Modify the equations so the target is flying <u>**away**</u> from the radar. Use the same speed as is used in the unmodified files. If you have trouble finding anything about velocity in the equations, look in the second paragraph of the attached document. I promise that the required changes to the equations will be minimal. Don't work too hard on this! Call me if you have a question. Save the file under the name " **_**_away.mcd ", where **_** are your last names.

2. Re-open the original "Application Exercise 1.mcd" file. This time modify the equations so that the target's opening velocity is twice what it was in the previous case. Save this file under the name " **_**_fast.mcd ".

3. Re-open the original "Application Exercise 1.mcd" file yet another time. This time modify the equations so that the target frequency is an integer multiple of the reference frequency. Save this file under the name " **_**_integer.mcd ".

4. Write up a brief description of **how Figures 3, 6, and 7 vary** with the different conditions. Include in your write-up possible **limitations** to this phasor analysis method that these three cases may highlight, **and what they mean** to radar designers and operators. An important consideration in your write-up should be that the <u>only</u> calculation that a digital radar's computer does is the one that produces Figure 7-- that's all it gets to see when it's making decisions about what to tell the pilot about a target. It's also important to understand that a target's closure speed is directly related to the frequency difference between the transmitted and received frequencies—we'll discuss this *Doppler Effect* in more detail in a few lessons.

5. Label your disk containing the three modified files with your two names, and turn in your disk and your write-up.

If you want to see the Mathcad file that is attached to this handout as it is displayed on the computer, it's on the website.

Again, this application shouldn't be too hard, but you should use this application along with the readings in *Hughes* to <u>really</u> understand the concept of phasors. If doing all of these things just don't make it click for you, ***<u>PLEASE</u>*** come see me for EI. The phasor concept is <u>critical</u> to understanding the next 18 or so lessons.

How Phasors Make Signal Processing Understandable

The following two graphs are time-domain representations (amplitude versus time plots) of the combination of two sinusoidal signals. Figure 1 shows the reference signal in red (the larger amplitude signal on the black-and-white printout), and the test signal in blue (the smaller amplitude signal on the top graph). Figure 2 shows the sum of the two signals in black. Note the oscillation of the envelope of the graph in Figure 2. This is called a "beat."

In terms of components in a digital radar, the large amplitude signal represents the local oscillator frequency generated by the super-accurate clock within the radar itself. The smaller signal represents the return from a target that is traveling <u>toward</u> the radar (hence it has a different, Doppler-shifted frequency). Figure 2 shows the signal that is actually sent to the digital processor, the sum of the reference and the target signals. Notice how complex the graph of the sum is.

$A_1 := 5 \quad A_2 := 3 \quad \omega_1 := 1 \quad \omega_2 := 1.1 \quad \text{timebins} := 5000 \quad i := 0, 1 .. (\text{timebins} - 1) \quad T_i := \frac{i}{25}$

$X_i := A_1 \cdot \sin(\omega_1 \cdot T_i) \quad Y_i := A_2 \cdot \sin(\omega_2 \cdot T_i) \quad Z_i := X_i + Y_i$

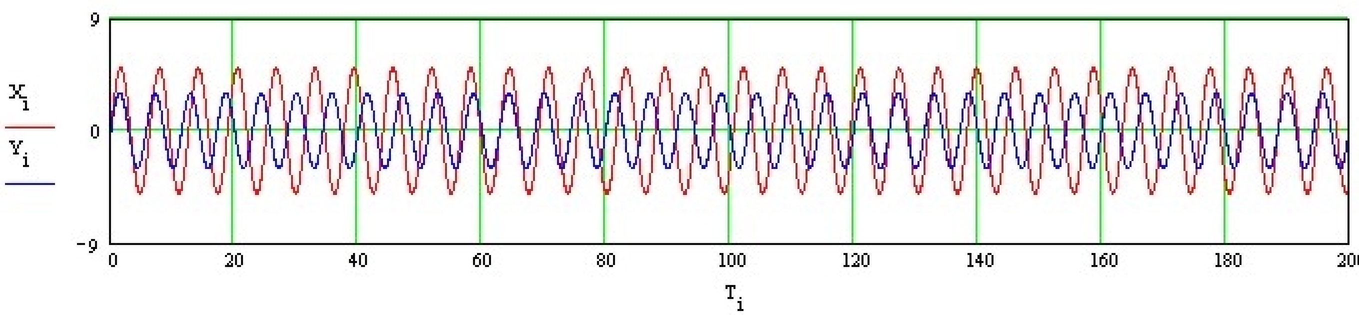

Figure 1: Two Sinusoidal Signals of Differing Amplitude and Frequency.

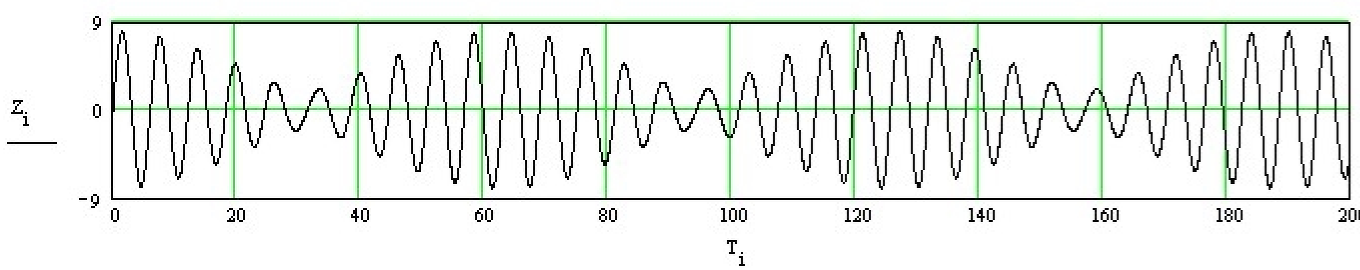

Figure 2: The Sum of the Two Signals in Figure 1.

The next two graphs are phasor representations of amplitude for the same two signals. Figure 3 corresponds to Figure 1 and Figure 4 corresponds to Figure 2. The sampling frequency is the same as the reference signal frequency, ω_1. The phase shift of the sampling frequency with respect to the reference frequency is ϕ. Note that the phasor amplitude oscillation has the same amplitude and frequency as the envelope oscillation of the time-domain plot (repeated here as Figure 5).

There are two primary reasons we want to look at the phasor representation of the signals. First, the graphs are a LOT simpler! Instead of a cosine function, the red local oscillator line (the top, dashed line in Figure 3) is a straight line at a constant amplitude of 5. This is due to the fact that the local oscillator has the same frequency as the sampling rate, so it always has the same relative phase (in this case, $\phi/2$). Since the smaller signal has a different frequency than the sampling rate, its phase when the sampler takes a "snapshot" of it varies with time. Sometimes the sampler sees it at its maximum, sometimes at its minimum, and sometimes its amplitude is zero. These results are reflected in the sinusoidal blue plot in the upper graph. Adding these two functions together, we get the greatly simplified graph in Figure 4. How much simpler? Compare the complexities of Figures 4 and 5! They both show the same signal, but Figure 4 is only sampled once every local oscillator period, while Figure 5 shows the entire signal.

The second reason we use this phasor representation is that a digital radar only samples the signal at certain times. The rest of the data is thrown away! It makes sense, looking at these phasor diagrams, to have your data collected at the same frequency as the local oscillator so that the radar's own reference signal doesn't complicate things too much.

$\phi := \frac{\pi}{2}$ $\quad j := 0,1..399$ $\quad t_j := 2\cdot\pi\cdot j$ $\quad x_j := A_1\cdot\sin(\omega_1\cdot t_j + \phi)$ $\quad y_j := A_2\cdot\sin(\omega_2\cdot t_j)$ $\quad P_j := x_j + y_j$

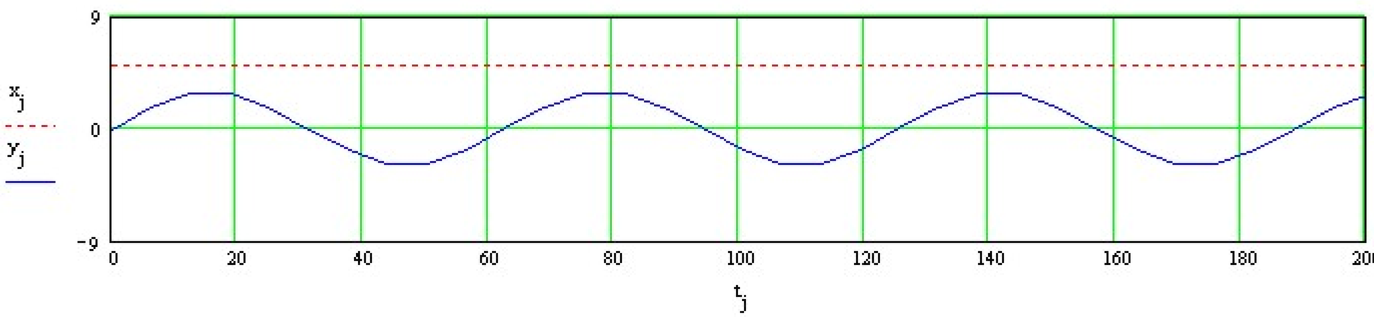

Figure 3: Phasor Representation of Figure 1.

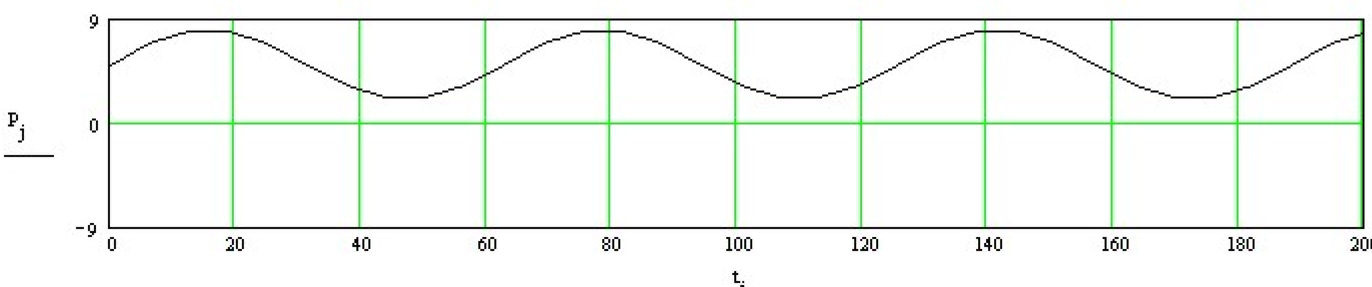

Figure 4: Phasor Representation of Figure 2.

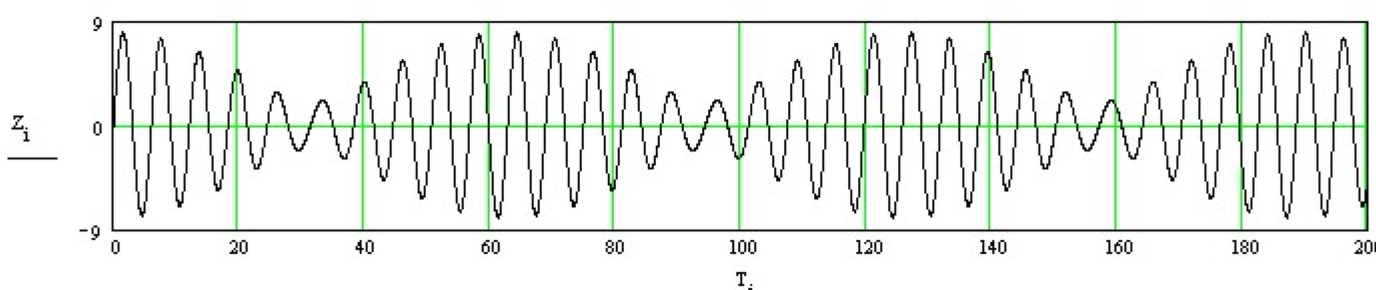

Figure 5: A Reprint of Figure 2, the Sum of the Signals in Figure 1.

So what does the phasor representation tell us? It is a quick way of seeing the beat frequency between the two component signals. The following two graphs are *Fourier transforms* of the time domain sum and the phasor sum. *Don't worry too much about the equations used to get the Fourier transformed graphs. Just realize that for our purposes, a Fourier transform takes a plot of amplitude vs. time and transforms it into a plot of relative amplitude vs. frequency, or a "frequency domain plot." Sometimes it's easier to analyze things in terms of time and sometimes it's easier to look at the component frequencies. In general, radar analysis, especially Doppler radar analysis, is much easier when done in the frequency domain.* Figure 6 is the transform of the time-domain sum in Figures 2 and 5. It tells us that the component frequencies that make up the signal are $\omega_1 = 1$ and $\omega_2 = 1.1$, exactly what we input. *The reason that it is a broadened peak instead of a single spike exactly at 1 and 1.1 is because we used a finite-length signal. Had we been able to analyze infinite sine waves of these frequencies, we'd have gotten perfect spikes, but we'll learn more about that later.* Figure 7 shows the transform of the phasor sum in figure 4. It tells us that there is a **strong** DC signal ($\omega = 0$) and a beat frequency of $\omega_{beat} = 0.1$. Note that the beat frequency is exactly $\omega_2 - \omega_1$! Thus, if we know ω_1, we can easily calculate ω_2. This is exactly how a digital radar calculates Doppler shifts. It merely does the Fourier transform of the phasor sum and subtracts off the frequency of the local oscillator!

$$f_{nyquist} := \left| \frac{\frac{1}{T_1 - T_2}}{2} \right| \qquad freqbins := \frac{timebins}{2} \qquad k := 0, 1 .. freqbins - 1 \qquad K := 0, 1 .. 250$$

$$Freq_K := K \cdot \frac{\left(\frac{1}{4 \cdot \pi}\right)}{200} \cdot 2 \cdot \pi \qquad freq_k := k \cdot \frac{f_{nyquist}}{freqbins} \cdot 2 \cdot \pi \qquad f := \overrightarrow{\left(\left| cfft(Z) \right| \right)^2} \qquad F := \overrightarrow{\left(\left| cfft(P) \right| \right)^2}$$

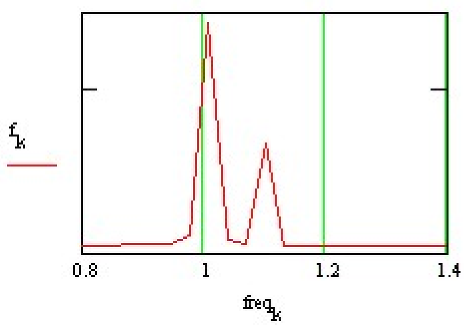

Figure 6: The Fourier Transform of Figure 5.

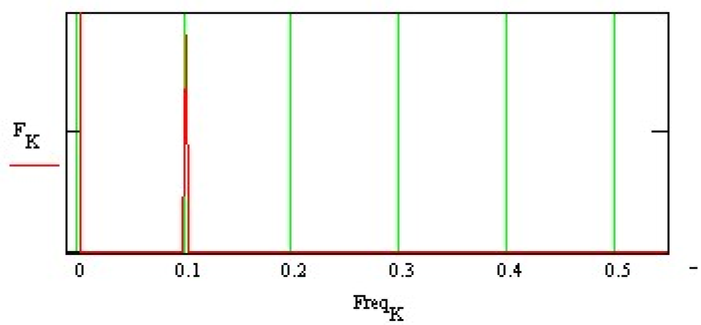

Figure 7: The Fourier Transform of Figure 4.

Finally, note that this result could have also been obtained mathematically from a basic trig identity, namely

$$\sin(\omega_1 \cdot t) + \sin(\omega_2 \cdot t) = 2 \cdot \cos\left[(\omega_2 - \omega_1) \cdot t \right] \cdot \sin\left[(\omega_2 + \omega_1) \cdot t \right]$$

For $\omega_2 - \omega_1$ = small, the cosine term on the right side varies slowly with time. Thus $\omega_2 - \omega_1$ is the frequency of the envelope of the sum of the two signals, or the beat frequency.

The moral of the story is that for a digital (digital = discrete sampling of data) radar, a simple way to analyze the frequency of an unknown signal is to mix that signal with a local oscillator having a similar frequency, sample the sum at exactly the local oscillator frequency, and determine the frequency of this sum. Simply adding this sum frequency to the local oscillator frequency will determine the frequency of the unknown signal.